REVIEWS

Elements of Compressible Flow. By F. Cheers. John Wiley, 1963. 166 pp. 36*s*.

Symposium Transsonicum. Edited by K. Oswatitsch. Springer-Verlag, 1964. 490 pp. DM 96.

F. Cheers complains with some justice that, for the purpose of an adequate appreciation of the principles behind the design of modern engines, textbooks on fluid mechanics usually have less than enough to say about compressible flow, that books on thermodynamics have less than enough to say about duct flow and that books on gas dynamics are too much oriented towards the needs of aircraft designers and missile technologists. He set himself the task of providing a textbook which makes it possible for a student to proceed to read the more advanced books and papers on engine design. The students he teaches in his own university are presumed to have a good knowledge of thermodynamics, and to have taken a course in classical hydrodynamics and to be attending a complementary course on incompressible fluids, including viscous flow, boundary-layer theory and turbulence, while following the course on which this book is based. It is clear that some such background is necessary for students attempting to use the book, which has little to say on these topics.

There are chapters giving a straightforward account in simple terms of one-dimensional unsteady and two-dimensional steady flow, including the properties of shock waves and Prandtl–Meyer expansions of relevance to flow in ducts. There follows a chapter on supersonic nozzles and diffusers indicating the types of flow that are possible. Next comes an account of numerical methods for problems of flow in two dimensions; the wavelet design method is used to calculate a nozzle for a supersonic wind tunnel. The method of characteristics is outlined as a means of dealing with, for example, flows with shock waves present, but this section is left rather in suspense, since it is neither written in generality nor actually applied to a specific problem.

The next chapter is concerned with the linearized equations of subsonic and supersonic flow. The limitations of the theory are indicated. There is a short chapter on real gas flows and transonic flow and another on measurements in incompressible flow. The book ends with a collection of almost wholly numerical examples on the work of each chapter.

Throughout the book charts are used and the author encourages their use. The mathematics is kept very simple. The text is well supplied with useful sketches, but what a pity there is not a single plate to illustrate the reality of a shock or expansion wave!

The Symposium Transsonicum met in 1962 at Aachen with the support of the International Union of Theoretical and Applied Mechanics. The meeting provided an opportunity for a distinguished group of workers in this field to give accounts of the present state of research and to draw attention to the many as yet unsolved problems in a subject which is still of the greatest physical importance, but which is rather neglected nowadays because sufficient advances have been made to allow transonic flight and unanswered questions relating to hypersonic motion and aerospace science appear to offer opportunities for more exciting research. The significance of this volume, splendidly edited by Professor Oswatitsch, has to be measured in terms of its contribution beyond what is already easily available. The two main texts are those of Guderley and Ferrari-Tricomi. The latter is written in Italian; the former appeared some years ago. The production of an up-to-date volume in which most of the articles are written in English may, then, be very much welcomed by the English-speaking readers of this Journal. This may, in fact, be one of the greatest merits of the volume since overall progress has not been rapid in recent years.

There are thirty-six contributions to the volume. They were presented at the Symposium in the course of ten sittings. Three of these were devoted to hodograph theory and singularities (papers by Guderley, Tricomi, Germain, Mackie, Laitone, Reyn, Růžička & Spaček, Rues), two to methods in the flow plane for plane and axisymmetric flows (papers by Zierep, Garabedian, Maeder, Tayler, Rotta, Spreiter, Hosokawa, Ferrari, Thommen), and one each to steady flows in three dimensions (papers by Küchemann, Fraenkel & Watson, Pearcey, Lock), internal flows and wind tunnels (papers by Berndt, Holt, Hall & Sutton, Romberg, Sandeman), unsteady flows and flutter (papers by Timman, Oswatitsch, Landahl, Stark, Destuynder & Chopin, Teipel), magneto-gas dynamics (papers by Tamada, Seebass) and one was called 'miscellaneous' at which Fiszdon read a paper on known applications of variational methods to transonic flow calculations and Imai gave a summary of Japanese research on transonic flow.

The contributions on hodograph theory and singularities are almost entirely review material, with the major contribution from Germain in an article, written in French, tracing the mathematical problems posed by the hodograph method and the methods so far developed to meet them. From the second group of papers one might pick out the contributions by Zierep (in German) on integral equation methods, Maeder's unifying account of the related methods associated with his own name and Spreiter's, and Spreiter's own lecture on his method of 'local linearization'. In this group there was also a comparison by Rotta (in German) of theoretical and experimental results for transonic flow over aerofoils, with some reflexions on the use of Spreiter's technique and the determination of the position of the shock wave on the body.

A distinctive contribution is that written by Küchemann, who outlines some of the three-dimensional flow problems of transonic type for slender wings and for lifting bodies in relation to the design of aircraft and other flying vehicles for hypersonic speeds. He has given an excellent discussion of the approach to design and of the appropriateness and deficiencies of theoretical work, with special reference to conical flows. A similar realism pervades the articles by Pearcey on flows with a local supersonic region embedded in a subsonic flow and terminated by a shock wave impinging on the surface of the aerofoil or wing, by Lock on the design of swept wingbody combinations for transonic speeds, and by Berndt on wall interference in transonic wind tunnels.

D. C. Pack

Perturbation Methods in Fluid Mechanics. By MILTON VAN DYKE. Academic Press, 1964. 229 pp. £2 16*s*.

The technique of deriving what one hopes to be an approximate solution of a hydrodynamic problem by omitting from the governing equations terms which appear to be 'small' in some sense is, of course, as old as the subject of hydrodynamics itself: Lamb's book abounds in persuasive phrases like 'the motion being assumed to be infinitely small' (for surface waves) and 'when the effects of inertia are insensible' (for Stokes flow). In the last 20 years, however, a slightly more critical view of this procedure has been advocated by leading research workers and adopted by many of the rest of us. It has become common to try to identify such an approximation with the first, or first two, terms of a series

$$\sum_{n=1}^{N} \delta_n(\epsilon) f_n(\mathbf{x}), \quad \text{where} \quad \frac{\delta_{n+1}(\epsilon)}{\delta_n(\epsilon)} \to 0 \quad \text{as} \quad \epsilon \to 0, \tag{1}$$

which should ideally be an asymptotic expansion of the exact solution $f(\mathbf{x};\epsilon)$; and, where possible, to calculate terms of higher order. Moreover, it has become clear that only in a minority of problems does a single expansion of the form (1) describe the exact solution with an error of only $o\{\delta_N(\epsilon)\}$ for all values of $\mathbf{x}$ in the domain D of interest. More usually the error of the series (1) exceeds $o(\delta_N)$ somewhere in D (examples are the failure of the inviscid-flow approximation near a solid boundary, and of the Stokes approximation at large distances from a body in a uniform stream), and when boundary conditions refer to this part of D direct calculation of the $f_n(\mathbf{x})$ becomes rash or impossible.

Fortunately, the masters of perturbation theory have not only recognized difficulties such as these, but have also devised techniques for overcoming them; and it is this subject—the diagnosis and cure of the disastrous non-uniformities often introduced by quite plausible schemes of approximation—which forms the main theme of the present book. Two methods receive most attention: the generalized boundary-layer technique often called the method of 'inner and outer' (but here of 'matched asymptotic') expansions, and associated with the names of Friedrichs, Latta, Kaplun, Lagerstrom and Cole; and the method of 'strained co-ordinates', which is related to the theory of characteristics for non-linear hyperbolic equations, and to work of Poincaré on ordinary differential equations, and is associated with the names of Lighthill and Whitham.

Both methods have had spectacular successes in yielding descriptions of flow fields (slow viscous flow past a circular cylinder, the far field of a supersonic projectile) which previously seemed anomalous or hopelessly difficult; both are by no means easy. For they demand either a real appreciation of uniform limits and of singularities, or exceptional physical insight; and, although valuable advances towards rigorous theories have been made by a number of mathematicians,† there are, as far as I know, no general theorems

† See references given in the book under review, and also the following papers, and references given in them: P. A. Fox, *J. Math. Phys.* **34**, 1955, p. 133; K. Nickel, *Arch. rational mech. anal.* **13**, 1963, p. 1; W. Wasow, *Arch. rational mech. anal.* **14**, 1963, p. 61.

which *a priori* guarantee success in applications of these methods to non-linear partial differential equations. Professor Van Dyke has wisely chosen to base his account mainly on examples, with strong appeal to intuition and to the physical meaning of each calculation, and it is a triumph of his book that these advanced techniques are thereby brought well within the reach of graduate students of engineering, science or mathematics.

The examples cover a wide range of aerodynamic problems and are splendidly done, in a tidy and systematic fashion. Uniform approximations are illustrated first by adding to the potential flow past a circle the effects of slight shear, slight distortion of the boundary, and slight compressibility. The methods of matched expansions and of strained co-ordinates are then introduced as alternative prescriptions for correcting thin-aerofoil theory near rounded and sharp leading and trailing edges in the case of incompressible flow, and far from the aerofoil in the case of supersonic flow; some simple ordinary differential equations are also considered at this stage. Next, boundary-layer theory is presented, with emphasis on the flat plate, for which the matched expansions are taken to a third approximation and optimal co-ordinates are introduced; this is followed by an account of the celebrated work of Kaplun and of Proudman & Pearson on the slow viscous flow past a sphere and a circle. Matched expansions are then used to produce a sparkling version of lifting-line theory which is more rational, goes further, and is simpler than the classical version, and to study the hypersonic flow past a blunted wedge; strained co-ordinates are used to find the shock position on a cone in slightly supersonic flow.

Few of these results can be justified with complete rigour, but all are self-consistent and highly plausible, and where the possibility of comparison with an exact solution arises, the author exploits it to maximum effect. Routine calculations are outlined clearly, and yet the text never becomes dull or plodding. There are valuable exercises at the end of each chapter which consolidate and extend the main material. There is also a great deal of ingenious advice on how to make shrewd guesses and on how to extract the greatest amount of numerical information from only a few terms of a series.

That smaller part of the book which deals with fundamental aspects of the techniques in question is less satisfactory, particularly for readers who expect precise definitions, unambiguous statements of general principles, and a clear distinction between proof, plausible inference, and wild conjecture. The imperfections here are due partly to the incomplete state of the subject, but some result from the author's free-and-easy approach. For example:

(i) We read on p. 83 that the general outer expansion

$$\text{‘}f(x,y,z;\epsilon) \sim \Sigma\, \delta_n(\epsilon) f_n(x,y,z) \quad \text{as} \quad \epsilon \to 0 \quad \text{with} \quad x,y,z \text{ fixed} \qquad (5{\cdot}8)$$

... is valid wherever the functions f_n are regular'. But in the prototype problem

$$\left.\begin{aligned} \epsilon\frac{d^2f}{dx^2} + \frac{df}{dx} = g(x), \quad f(0;\epsilon) = 0, \quad f(1;\epsilon) = 1, \\ g(x) \text{ regular for } O \leqslant x \leqslant 1, \quad \epsilon > 0, \end{aligned}\right\}$$

(a particular case of which has been considered on p. 79), f has an outer expansion which is not valid in any sense near $x = O$, although the f_n are regular there. The outer expansion in problems of flow at large Reynolds number provides another counter-example.

(ii) The definition on p. 89 of an *overlap domain*, namely 'where both the inner and outer expansions are valid', is by itself too vague to be grasped or applied, and is followed by nothing more explicit in terms of limits. (Kaplun and Lagerstrom's original definition may perhaps have looked difficult, but it was wholly precise and immediately applicable). Moreover, when we meet the term overlap domain again on p. 92, 13 lines down, we face the following choice: either its meaning is not that implied on p. 89, or the statement that its existence is assured by Kaplun's extension theorem is not true in general. For there are many quite ordinary functions whose inner and outer expansions do not overlap to specified orders in the sense of Kaplun and Lagerstrom.

(iii) The following 'asymptotic matching principle' is stated on p. 90 and applied throughout the book:

'The m-term inner expansion of (the n-term outer expansion)

= the n-term outer expansion of (the m-term inner expansion). (5.24)

Here m and n are any two integers.'

Consider the function

$$f(x;\epsilon) = 1+\frac{1}{\log\epsilon}\log(x+\epsilon), \quad O \leqslant x \leqslant 1, \quad \epsilon > 0,$$

whose outer expansion

$$1+\frac{1}{\log\epsilon}\log x+\frac{\epsilon}{\log\epsilon}\frac{1}{x}+\dots$$

fails near $x = 0$. The appropriate inner variable is $X = x/\epsilon$ and the inner expansion is

$$2+\frac{1}{\log\epsilon}\log(X+1).$$

Clearly, if $m = n = 1$, the left-hand side of (5.24) is 1, while the right-hand side is 2; thus the principle does not always hold. It is true that we are warned on p. 5 (for a particular case) and on p. 201 that terms in $\epsilon^n \log\epsilon$ and ϵ^n 'must be regarded as together constituting a single step in the process of successive approximation'; but since no explicit restriction is placed on (5.24), and since in the case of slow viscous flow past a circle it appears to be correct and necessary to separate terms in ϵ^{-1} and $(\epsilon\log\epsilon)^{-1}$ [see (8.41), p. 162], we are left in a state of confusion. Moreover, the same weakness of (5.24) leads to a difficulty in the problem of flow past a biconvex aerofoil (p. 70 and exercise 4.5), where warnings about logarithms are again ignored, and it is not made clear in that case how the difficulty can be resolved without appeal to the exact solution.

In addition to this kind of looseness in the mathematics, there are one or two lapses regarding aerodynamics. The fact that 'the hyperbolic equations of inviscid supersonic motion become elliptic in the linearized theory of conical flow' is cited on p. 42 as an example of a perturbation method spuriously changing the type of the governing differential equations. But there is nothing spurious here: inside the relevant characteristic cones (and there only) both the non-linear and linearized equations of *conical* supersonic flow are elliptic—for an obvious physical reason.

Despite these criticisms, my overall impression is one of a spirited and mainly successful book which not only conveys the author's enthusiasm for modern perturbation problems, but also lays before us the considerable fruits of his ingenuity and wide experience. Readers of this Journal who rely primarily on intuitive reasoning will, I imagine, welcome the book whole-heartedly; those who value the processes of analysis may treat it with greater reserve, but should be grateful for an attractive display of systematic approximation procedures applied to a variety of difficult examples. L. E. FRAENKEL

Design and Performance of Centrifugal and Axial Flow Pumps and Compressors. By A. KOVATS. Pergamon Press, 1964. 468 pp. £5. 5*s*.

The economics of power generation becomes increasingly important as the demand for power supplies increases. Important items in the efficient operation of a power generation plant are the turbomachines, in which the compressors and pumps are particularly sensitive to faulty design of the passages. Over the past two decades much of the large increase which has taken place in basic research in fluid mechanics can be applied to widen our understanding of the flow through such plant. It is important that this should be put in a form available to the designer. The author seeks to do this using a unified treatment of compressible and of incompressible flow. This is in line with the increasing tendency to bring together the field of thermodynamics and that of the fluid mechanics of incompressible flows, so that the behaviour of different types of turbo-machines can be studied from the same basic principles. It seemed, however, to the reviewer that the value of this book was not in its exposition of the fundamentals but in its survey of a wide range of research quoting results in a form suitable for application to centrifugal and axial flow pump and compressor design.

Topics covered include losses due to boundary layer behaviour, secondary flows, shock waves and cavitation; velocity distribution in blade passages; operation and performance of pumps and compressors; pumps for special duties; design of impellers and arrangement of mechanical details. Due to the limitation of space in a single volume the discussion often seemed inadequate for the student of fundamentals, but lists of references are given for more comprehensive study of the topics dealt with. H. G. RHODEN

The Gulf Stream: A Physical and Dynamical Description. By HENRY STOMMEL. Second edition. 248 pp. $6.00.

The first edition of this book was reviewed in this *Journal* in 1960 (vol. 9, p. 632). The second edition differs in two respects; it costs rather more and it contains a final long additional chapter entitled 'Recent Developments'. This chapter is a review of work published on the Gulf Stream since the first edition of the book was written.

New observational material is first discussed. This includes the results of the multiple ship survey of 1960, the deep current measurements of Swallow and

Worthington, Volkman and Barrett and new information about the fluctuations and meanders in the stream from the work of Webster. Though the situation is far from clear it appears that there is a deep countercurrent south of Cape Hatteras whereas the flow at depth east of Hatteras is in the same direction as the surface flow. A theory of Warren, based on this latter fact, predicts the meander patterns with satisfying accuracy. Computations by Webster of momentum transfer south of Hatteras suggest that the eddies supply momentum to the mean flow.

The theoretical developments are largely concerned with western boundary currents. The work of Morgan, Veronis and Moore receive particular mention and Bryan's numerical model is described in considerable detail. There is also a brief discussion of recent theories of the oceanic thermocline and the associated thermohaline circulation.

In the final section of the chapter, entitled 'Recommendations', it is suggested that large-scale quasi-geostrophic eddy processes play an important dynamical role in the vorticity balance of the ocean interiors, thus casting doubt on the validity of the basic Sverdrup relation between the local curl of the mean wind-stress and the vertical integral of the meridional component of velocity. Support for this idea comes from several sources but notably from the *Aries* data which revealed long period eddies with r.m.s. amplitude two orders of magnitude greater than the expected mean velocity. The author draws an analogy here with the development of the theory of the general circulation of the atmosphere. The idea that the large horizontal eddies play an essential part there was put in a semi-quantitative form by Jeffreys in 1926 (*Quart. J. Roy. Met. Soc.* **52**, p. 85) though it was not until after the Second World War that sufficient upper air data were readily available for more precise quantitative estimates of the horizontal transfers to be made by Priestley (*Quart. J. Roy. Met. Soc.* **75**, 1949, p. 28). It seems that our knowledge of oceanic circulations has now reached a stage where semi-quantitative estimates are possible but we may have to wait some years before there is sufficient data for more precise estimates of the horizontal eddy transports to be made.

F. K. Ball

Fluid Mechanics. By D. Gilbrech. Wadsworth Publishing Company, 1965. 562 pp. £5. 5*s*.

Fluid Mechanics for Civil Engineers. By N. B. Webber. E. and F. N. Spon, 1965. 340 pp. £3. 7*s*. 6*d*.

These latest additions to the already embarrassing number of University text-books on fluid mechanics can at least claim to be slightly different. The former shows a certain originality of approach, and the latter is intended to bridge the gap between general text-books and advanced works on civil engineering hydraulics.

Professor Gilbrech's book provides a clear and simple introduction, but is so concerned with the fundamentals of incompressible and compressible flow, that it omits many topics such as free surface flows, machines and surges, which undergraduates at British Universities normally require in their first and second year. Perhaps its best feature is the large number of simple but searching problems together with answers. Many of these questions, for example, those concerning other planets having different atmospheric and gravitational conditions, are bound to exercise the student's understanding of fundamentals. The many excellent diagrams, perhaps because of their almost Victorian style, provide unusually strong emphasis to the clear explanations given. However, despite its originality, it is unlikely to be popular with engineering students in Britain because of its relatively high price and somewhat restricted coverage.

Webber's book is claimed to be of particular interest to civil engineering students requiring a general introduction to the subject. It certainly has a civil bias and omits compressible flow, but, on closer examination, it does not differ from existing student text-books on fluid mechanics as much as one might have expected. Nevertheless, it goes into rather more detail than usual on pipe-network problems and on free surface flows in channels, rivers, and in hydraulic structures. It is written from the viewpoint of an experienced civil engineer, now a University lecturer, and contains many small additional items of civil engineering interest. It is particularly well illustrated with photographs of model and full scale hydraulic works.

In places, accuracy has been sacrificed to simplicity; for instance, the statement that a fluid 'differs from a solid in that it suffers deformation due to shear stress' is misleading, although many will realize what the author is trying to say. Again, the simplification that 'a vapour is a gas which is near the liquid state' depends upon one's interpretation of 'near'.

However, despite a few trivial oversimplifications, the general approach is good and provides a sound introduction to the subject, together with useful bibliographies for further reading. The use of momentum correction coefficients to allow for velocity variation is particularly welcome. The book is relatively expensive compared with many similar text-books now available, perhaps because of the large number of excellent plates and figures.

S. F. Hutton

SHORTER NOTICES

Pumps and Blowers: Two-Phase Flow. By A. J. Stepanoff. John Wiley and Sons, 1965. 316 pp. $12.50 or £4. 14*s*.

A collection of articles on advanced topics concerned with the design of pumps and blowers, the pumping of cavitating liquids, transport of mixtures of liquid and gas, or solid suspensions such as paper pulp, along pipelines, and the gravity flow of granular material. The author's papers in these fields are already known to specialists but this book is useful in bringing together information for the designer. Part 1 suggests improved cavitation parameters to allow for thermal properties of fluids and considers dis-similarity rules and detailed design features for pumps and blowers. Part 2 reviews the problems of two-phase flow and bulk flow. Bibliographies of literature up to about 1960 are provided with each chapter. Much of the material is inevitably semi-empirical, but is nevertheless of practical value to the specialist designer and user of machines and systems to cope with difficult two-phase conditions.

Ionized Gases. By A. VON ENGEL, 2nd edition. Oxford University Press, 1965. 325 pp. £2. 15*s*.

For this second edition, the general plan and scope are similar to those of the original edition (1955). There are many changes in detail, however, especially in the chapter on the production of charged particles, and a number of additional diagrams, tables and references. The book is concerned exclusively with physical processes, such as ionization, recombination and charge transfer, and observational matters like the glow and arc discharges. This is a rather difficult field to survey attractively, and readers will be grateful to the author for producing a lucid book of reasonable length. No attempt is made to cover the perhaps more fashionable subject of dynamical processes like plasma confinement, wave propagation and instabilities.

Advances in Theoretical Physics, vol. 1. Edited by K. A. BRUECKNER. Academic Press, 1965. 323 pp. £4. 16*s*.

The editor writes in the preface: 'It is the intent of this serial publication to present review articles in the major fields of theoretical physics, including elementary particle theory, nuclear structure, hydrodynamics, geophysics, and numerical methods of mathematical physics, to give specialists ready access to important advances in other fields. We hope that *Advances in Theoretical Physics* will provide a unifying element which may, to some extent, offset the present fragmentation of theoretical physics'. This is a laudable objective, notoriously difficult to achieve. The titles and authors of articles in this first volume of the series are as follows:

'The theory and application of the Padé approximant method', by G. A. Baker;

'Theory of the giant dipole resonance', by W. Brenig;

'The optical model', by A. L. Fetter and K. M. Watson;

'Hydromagnetic equilibrium and stability', by J. M. Greene and J. L. Johnson;

'The heavy ion transfer reaction', by K. R. Greider;

'Elastic scattering of electrons by atoms', by M. H. Mittleman.

Tenth Symposium (International) on Combustion. The Combustion Institute, Pittsburgh, 1965. 1488 pp. $35.

These enormous volumes on combustion keep on appearing each year, and the latest is as heavy and as difficult to review as its predecessors. Those who are involved in the business of combustion will no doubt have been to the symposium and will have their own copies; and those who are not need only be told that this particular volume contains cumulative subject and author indexes for the papers published in all 10 volumes of proceedings of the combustion symposia.